

Foundations of Logic

Instructor

Dag Westerståhl

Professor, Tsinghua University Department of Philosophy, School of Humanities

Jin Yuelin Professor at Tsinghua; Professor of Theoretical Philosophy and Logic at Stockholm University; Member of the Royal Society of Arts and Sciences in Gothenburg and of the Royal Swedish Academy of Sciences. He is also on the Advisory Board of the Joint ILLC and Tsinghua Research Center in Logic. His main research interests are logic, formal semantics, and philosophy of language. A central theme is generalized quantifiers, studied both from a model-theoretic and a linguistic point of view.

About this course

The course presents basic facts which lie at the foundation of modern logic. First, we show that first-order logic itself is complete, in the sense that there is a simple system of axioms and rules (that we present) which derives all logical consequences of any given premises. Then we look at theories expressed in this language. After a glimpse of what one can say about models of such theories, we present Gödel's famous incompleteness theorems: in any axiomatic theory, which is free from contradiction and contains a bare minimum of arithmetic, there are true statements which cannot be proved in the theory. The methods used to show this lead to other important facts, such as Tarski's theorem on the undefinability of truth, and the fact that the freedom of contradiction of such a theory cannot be proved in the theory itself (the second incompleteness theorem). We discuss the philosophical import of these results, but our main focus is on how they are established. Finally, we show that first-order logic is undecidable: there is no effective method

(computer program) which can decide, for any premises and conclusion, if the conclusion follows logically from the premises or not.

Reference textbook: Dag Westerstahl, Basic Metalogic: Completeness, Incompleteness, Computability, Stockholm University

Prerequisites:

It is assumed that students have some understanding of the elementary content of logic, but there is no strict requirement.

Syllabus

1 Background-1

- Propositional Logic
- First-Order Logic

2 Background-2

- Natural Deduction
- A Hilbert system

3 Completeness

- Completeness of propositional logic
- Completeness of first-order logic

4 Model Theory

- Model Theory-1
- Model Theory-2

5 Incompleteness-1

- Overview
- Primitive Recursive Functions and Relations

6 Incompleteness-2

- Peano Arithmetic
- Definable in PA

7 Arithmetization

- Arithmetization

8 Incompleteness-3

9 Incompleteness-3

10 Final Exam